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UNA INGENIERÍA COOPERATIVA EN ACCIÓN: PLANTEAMIENTO Y RESOLUCIÓN DE PROBLEMAS

A COOPERATIVE ENGINEERING IN ACTION: PROBLEM POSING AND SOLVING

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Resumen

En este artículo presentamos los resultados de un trabajo entre investigadores y profesores en el marco de la ingenieía cooperativa. La ingeniería cooperativa se basa en principios que se entrecruzan para pensar e imaginar la acción, derivados de la experiencia. El primero es el de la simetría entre los distintos miembros del colectivo. El colectivo de profesores e investigadores participa en una relación epistémica cooperativa que postula la búsqueda de la simetría epistémica en el diálogo ingenieril. Profesores e investigadores tienen que trabajar juntos para producir una obra común, una secuencia didáctica de 18 lecciones de planteamiento y resolución de problemas.

Se trata de la primera fase de un programa de investigación que consiste en diseñar y aplicar una secuencia de planteamiento y resolución de problemas para alumnos de 7 a 9 años (2º y 3º de primaria) durante un periodo de dos años. El proceso de ingeniería está diseñado para comprender y mejorar la secuencia de instrucción con el objetivo de aumentar tanto la comprensión como la eficacia. Examinaremos los resultados del trabajo de este grupo durante esta fase inicial de la investigación. La secuencia pretende vincular el planteamiento y la resolución de problemas. Alumnos y profesores se familiarizan con representaciones como los cubos, la recta numérica, la caja y los escritos matemáticos. Estos hábitos se basan en la imitación. Primero, el profesor muestra a los alumnos cómo utilizar estas representaciones. Los alumnos hacen lo mismo al principio, y después son capaces de representar su propio problema. El colectivo de profesores e investigadores ha optado por que el profesor y sus alumnos utilicen la categorización de problemas en la clase. Cada alumno es capaz de presentar y resolver un problema explicando su categoría. Para recordar lo que han hecho los alumnos, la profesora les ayuda a elaborar un instrumento de repertorio. Este instrumento puede ayudarles a plantear y resolver el problema. La secuencia se divide en seis unidades, cada una de las cuales se organiza del mismo modo en torno a una categoría (situación no problemática, representaciones, ejemplos trabajados, planteamiento del problema, instrumento de repertorio, ...). De este modo, el profesor y los alumnos se familiarizan con cada categoría de problemas. El planteamiento de problemas y la categorización se elaboran y entrelazan para que los alumnos comprendan la estructura conceptual de los problemas y puedan resolverlos mejor.

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Como resultado, compartimos la relación en ingeniería cooperativa como una relación de indagación sobre los conocimientos a enseñar y también sobre las prácticas de enseñanza de estos conocimientos. Cuanto más se pone en práctica la secuencia, más explícita y compartible se vuelve. Basándonos en estas conclusiones iniciales, presentaremos algunas perspectivas para los próximos pasos de esta investigación. Los profesores y los investigadores no sólo deben desarrollar una comprensión compartida de la secuencia, sino que ahora deben hacer explícita su comprensión a otros profesores. Se trata de cambiar fundamentalmente la comprensión que tienen los profesores de los conocimientos que están en juego en la secuencia de planteamiento y resolución de problemas..

Palabras clave: Planteamiento y resolución de problemas; escuela primaria, ingeniería cooperativa; representaciones; imitación.

Abstract

In this paper, we present results of a work between researchers and teachers within a cooperative engineering framework. This is the initial phase of a research program that involves writing and implementing a sequence for posing and solving problems for pupils aged 7 to 9 (2nd and 3rd grade) over a two-year period. The engineering process aims to understand and improve the teaching sequence, with the goal of enhancing both understanding and effectiveness. We will examine the result of the work of this group during this initial phase of the research. The sequence aims to link problem posing and problem solving. As a result we share the relationship in cooperative engineering as a relationship of inquiry about the knowledge to be taught and also about the practices for teaching this knowledge. Based on these initial results, we present some perspectives for the next steps of this research.

Keywords: Posing and solving problem; primary school, cooperative engineering; representations; imitation.

1. Introduction

In this paper we propose to describe a teaching sequence developed by the DEEC research (Determining the Effectiveness of Controlled Experiments in Teaching and Learning Mathematics), funded by the French National Research Agency (NRA). The aim of this research is to identify the determinants of the effectiveness of a teaching sequence in mathematics (Cai et al., 2015; Kilpatrick, 1987).

This teaching sequence is designed within the framework of cooperative engineering involving teachers and researchers. This teaching sequence consists of a posing and solving problem sequence for pupils (2nd and 3rd grade, aged 7 to 9), in France. The engineering process thus aims to understand the teaching sequence in order to improve it, and by improving it, to understand it better (Sensevy, 2012).

In a first part, we present a theoretical and methodological framework for cooperative engineering (Joffredo-Le Brun et al., 2018; Morales et al., 2017; Sensevy & Bloor, 2019; Sensevy et al., 2013). This includes conceptual elements related to representations (Fischer et al., 2019; Joffredo et al., 2017) and problem posing (Cai, 2022).

In a second part, we describe the teaching sequence, which is based on an unproblematic situation, by proposing different examples. We highlight on the choices made by the cooperative engineering collective, shared by each of its members. We then present a tool, the repertoire instrument, which reflects the progress of the students' knowledge in the classroom, and the understanding of the collective members who implement it.

Finally, the concluding section looks ahead to the next step of this research, in particular on the role of cooperative engineering members in the dissemination of such a didactic device and the perspective of analysis which will be carried out linking evidence-based practice to practice to improve the effectiveness of this device and determine the reasons for this effectiveness (Bryk, 2015).

2. Theoretical framework

2.1. Cooperative engineering

Cooperative engineering developed in Joint Action Theory in Didactics (JATD) (Sensevy, 2014; Sensevy & Bloor, 2019) is a process that associates education professionals, teachers and researchers. They have to cooperate in order to produce a common work. Its aim is to produce a didactic device, i.e. a set of classroom lessons based on working hypotheses. In our study, we designed a teaching sequence of 18 lessons in problem solving. This engineering involves 3 trainers, 16 teachers implementing the lessons in their classrooms, and 6 researchers (Joffredo-Le Brun et al., 2018; Sensevy & Bloor, 2019; Sensevy et al., 2013). The development of this teaching sequence is based on a collective inquiry (Dewey, 1938). The aim is to build cooperative knowledge of both the knowledge to be taught and the teaching sequence (description, analysis for understanding and mastery, etc.). Four phases are essential to its operation. Firstly, collective studies knowledge per se, without any didactic implications. Next, the sequence to be implemented in the classroom is designed. An analysis is then produced with regard to the working hypotheses implemented, the culture and the powers of the knowledge that this implementation is intended to “concretize" (Sensevy, 2021). A second implementation is then put in place to produce a textualisation of this didactic device.

Cooperative engineering is based on principles that interlace to "think and imagine action, derived from experience" (Collectif Didactique pour enseigner [CDpE], 2024). The first is that of symmetry between the different members of the collective. The idea is that in order to be viewed a priori as equally able to propose adequate ways of acting or relevant ways of conceptualizing practice in the elaborated design (Sensevy & Bloor, 2019). They participate in an epistemic cooperative relationship, which postulates striving for an epistemic symmetry in the engineering dialogue.

However, this quest for symmetry does not ignore the issue of differences between agents. These differences are not based on the status of those who know something versus those who do not. Each member of the group is responsible for putting forward their own point of view, based on their own experience. Teachers and researchers work together to construct the ends and means, which are developed through dialogue between the practical and conceptual points of view. Through this search for shared explicitness, everyone becomes involved in a game of offering and demanding reasons (Brandom, 1994). In this way, every member of the engineering team can appropriate and respond to the questions posed, understand and establish a first-hand relationship with the design logic, whether 'practical' or 'theoretical'. The posture of engineer is therefore achieved in the engineering dialogue where at certain moments in the different stages of designing the didactic device, teachers and researchers play the role of engineer, each bringing theoretical and/or concrete means to solve a practical problem while retaining their epistemic specificity. There are therefore local and occasional zones of indistinguishable role between researchers and teachers where each member of the engineering team plays the role of didactic engineer. The posture of engineer will take concrete form in the development of a sequence based on posing and solving problem.

2.2. Problem posing

Problem posing is one fundamental aspect of mathematical and scientific activities (Kilpatrick, 1987). The most essential aspect of scientific inquiry is knowing how to formulate a problem before attempting to solve it. Thus, an important teaching objective could involve engaging students in the process of problem posing, which is recognized as a significant intellectual activity in mathematics education (Cai et al., 2015). This could require a notable shift: teachers must share mathematical authority in the classroom, empowering students to generate their own problems. Contemporary research (Felmer & Perdomo-Díaz, 2016; Singer et al., 2015) highlights the value of students problem posing. Singer and Moscovici (2008) described a classroom cycle that includes problem posing as an extension of problem solving. Ellerton (1986) situated the process of problem posing within a broader context. Cai and Hwang (2020) analyzed specific instances of problem posing in various contexts, focusing on how teachers assist their students in formulating their own problems. The objective of these researchers is to understand the implications of teaching through problem posing. Based on Cai's perspectives (Cai et al., 2015), we define problem posing as the following specific intellectual activity: the team of teachers and researchers elaborate mathematical problem posing situations for students to formulate problems using a detailed organizational approach. This approach ensures that students will pose mathematical problems grounded in real-life scenarios.

To design this teaching sequence of posing and solving problem, the engineering team interacts. For this, they rely on the description and understanding of the different successive implementations in classes and hypotheses of common work in a cooperative epistemic relationship. This dialogue focuses mainly on the use of representations in the posing and solving of arithmetic problems.

2.3. Representation

It is important to specify what we mean by r*epresentation*. Unlike a cognitive approach, it is not the "result of deep mental activity involving a whole set of processes aimed at processing information about our environment provided by our sensory organs" (Julo, 1990). The representation, as we understand it, is also not linked to a cognitive process involving the transition from a representation of an object to another representation of that object (Duval, 2007). We use the term *representation* in a concrete and pragmatist perspective, based on Brousseau's proposition (Brousseau, 2004, p. 241): 'The term 'representation' refers to the action of 'rendering present again' and its result '. Each problem is represented in different ways (Fischer et al., 2019). We give now an example.

During the sequence, students are asked to talk about 20 students in the class, 13 students who eat at the canteen and 7 students who eat at home, as an unproblematic situation. This situation is not necessarily representative of their own class, but it is significant for students who know what a class is and which students do and don't eat at home. Students have to manipulate cubes. The cubes represent the 13 students in the class who eat in the canteen and the 7 students who eat at home (see fig. 1).

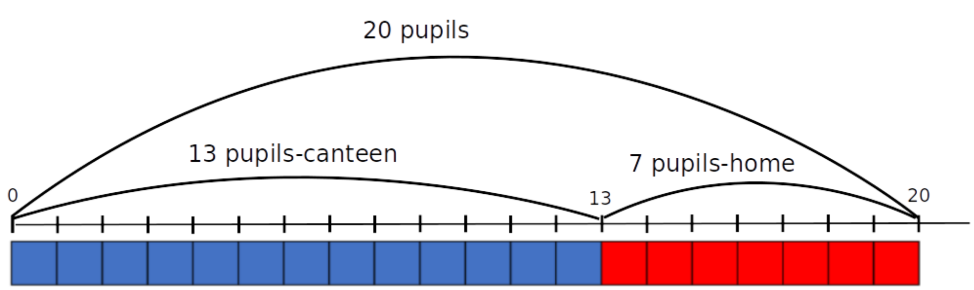
Figure 1

7 pupils-canteen and 13 pupils-home

Putting the cubes together means counting the total number of students in the class, namely 20 students in the class. This concrete, material representation of two collections will be translated into mathematical terms. In fact, this situation can be represented by the following calculation: 20 = 13 + 7. Furthermore, if we decide to take only the 7 cubes, we can say that the 7 students who eat in the canteen are the students in the class excluding the students who eat at home. Thus, we can write: 7 = 20 - 13. Similarly, as soon as we see an expression such as 20 = 13 + 7, we can take cubes and represent a situation. Moving back and forth between text, cubes, mathematical expressions, and/or schema will help develop a real understanding of representations, as we have conceptualized them (see fig. 2).

Figure 2

The number line and the cubes



As we noticed before, each member of the cooperative engineering is responsible for putting forward their own point of view, based on their own experience, in relation to representations. These different representations, which are discussed in the group, are presented to the class by the teacher.

We want to highlight a choice about the different categories of arithmetic additive problems and multiplicative problems (Vergnaud, 1982; Vicente et al., 2022). The problem showed in the previous example can be seen as a *representation* of the category of the combine problem. This example is *rendering present again* all what is done in the classroom, manipulating the cubes, writing mathematical expression, representation on the number line.

2.4. Research question

The engineer's role makes possible to consider a "space" shared by the collective in writing the sequence of posing and solving problems. This collectively constructed lesson sequence is implemented by some of the teachers in the collective. How is this sequence structured to promote a shared understanding of the mathematical problems within the class? How could the representation of the categories be shared in the class and in the collective of teachers and researchers?

3. The work in the cooperative engineering

We will investigate these questions using a cooperative engineering approach. The aim of this research project is to identify the determinants of the effectiveness of a teaching sequence in mathematics involving the posing and solving of arithmetic problems (Cai, 2022; Kilpatrick, 1987). The team is also interested in the ways used to support its implementation. As we said before the sequence has been developed by the collective of teachers and researchers. We have to understand that it is the result of many attempts in the classroom with teachers, who design it thanks to the collective. We don't develop the genesis of the sequence. We only present the state of progress of the problem posing-solving sequence.

3.1. The sequence

3.1.1. Introduction

The sequence is organized in 18 one-hour sessions, spread over 6 weeks. It covers additive problems such as combine, compare, change problems (Vergnaud, 1982; Vicente et al., 2022) thanks to 3 units (9 sessions), and also, multiplicative problems such as rate (ibidem.) thanks to 3 other units (9 sessions). The work on the categorization of problems is one of the main elements in the organization of the sequence and is at the heart of the collective dialogue. problems are worked on by category. The sequence begins with additive problems (combine, comparison, change problems) and ends with multiplicative problems. These terms are used in the collective of teachers and researchers, and in the classroom with students. As the sequence progresses, the different categories are revised, so that students are able to create or solve problems in different categories. In this way, problem categorization helps students to pose and solve real-life problems (Athias & Sensevy, 2023). For each problem category, the classroom implementation is structured in the same way, which we describe below with an example.

3.1.2. Organization of a problem category: an example

a) The unproblematic situation

Mathematics can be seen as a tool for representing the world. Mathematical symbols (numbers, operations) account for possible actions in this world. An unproblematic situation consists in the description of a portion of reality containing mathematics. The primary aim of the unproblematic situation is to start from what the student knows, from what he understands well, without being asked a school question - those questions for which the teacher already knows the answers.

The choice that has been made in this research DEEC is to work on unproblematic situations. This work integrates the understanding of the situation as well as the language aspects and the categorization of mathematical problems. We now show two examples of unproblematic situations.

Example 1: There are 20 pupils in a class. 13 eat in the canteen. 7 eat at home.

Example 2: A pen weighs 8g. A notebook weighs 22g. The notebook weighs 14g more than the pen. The pen weighs 14g less than the notebook. The difference between the weight of the pen and the weight of the notebook is 14g.

In these two examples, the goal is for the students to clearly understand the situation, that there is nothing to search for and that everything is known. The questions "How many pupils are there in the class?" or "How much does the pen weigh?" do not require a calculation; the answer is in the text. Similarly, these two problem-based situations are the basis for language exchange. Expressions are used by the teacher or the students, such as "there are 20 pupils in the class" or "the pen is lighter than the notebook; the pen weighs less than the notebook; the notebook is heavier than the pen"... These expressions can be considered as situational jargon (Jameau & Le Henaff, 2018). They are also used to introduce words such as "the whole" (e.g., 20 pupils), "the part" (e.g., 13 pupils-canteen), "the big compared" (e.g., notebook weigh), which can be considered as second-order jargon, categorical jargon, generic to some situations... These expressions (those of specific jargon as well as those of second-order jargon) will then be taken up again and explored in more detail at other times.

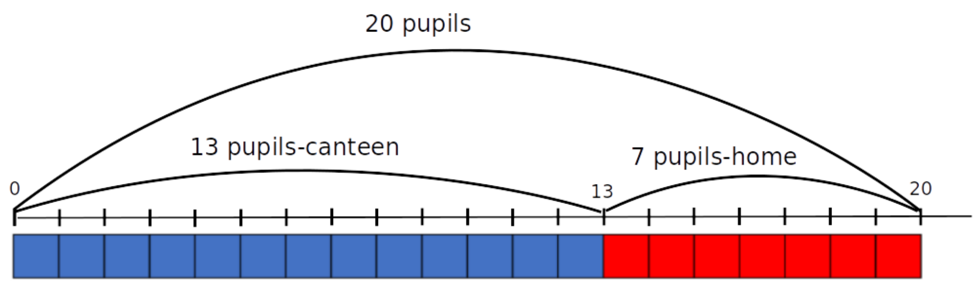
b) Working by example and imitation

The effect of the worked example has been extensively studied (Sweller & Cooper, 1985). Experiments have shown that worked examples require less time to process as they are studied, and that subsequent problems similar to the initial ones are solved quickly. A single worked example per teaching domain is unlikely to produce the effect of the worked example (Sweller, 2006). After studying a worked example, students need a problem to enable them to know whether they have learned. It is precisely the follow-up to a worked example that should encourage students to actively process it, for example by proposing an imitation of the worked example. In the field of education, Verba and Winnykamen (1992) suggested that imitation is the intentional use of an other's observed action as a source of information to achieve one's own goal. More broadly, imitative play is seen as an essential element in the transmission of culture (Heyes, 2016, 2023).

The choice made within the research DEEC is based on this imitation. It is organized as follows. The example is initially carried out by the teacher. The representations (the cubes, the number line, the number box... see below) are introduced using an example. Over time, these representations are taken up by the students, who gradually become familiar with them. An example worked on in this context is one of problem posing or solving, based on a situation that has not been problematic. This example is shown by the teacher. Let's take the previous example: there are 20 pupils in a class. 7 pupils eat at home (pupils-home). 13 pupils eat in the canteen (pupils-canteen). The teacher shows how this unproblematic situation can be represented in different ways. The first way is a number line based on the blue and red cubes (see fig. 3).

Figure 3

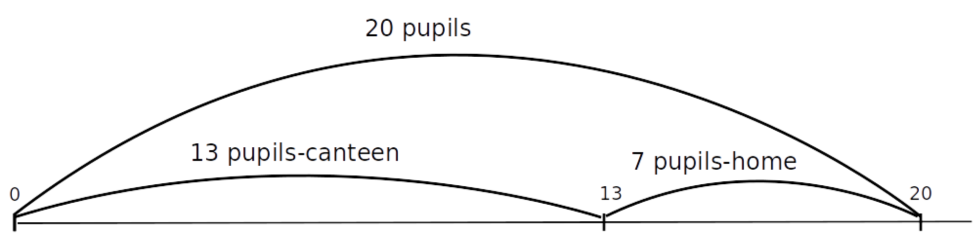
The number line and the cubes[1]



At the same time the teacher presents another way to represent this unproblematic situation: the cubes and the graduations on the number line have disappeared. The 0, 13 and 20 markers remain (see fig. 4).

Figure 4

The cubes and the graduations have disappeared



This same situation is represented in an other way: the situation box (see table 1). The 20 pupils in the class are distributed as follows: 13 pupils eat at the canteen and 7 pupils eat at home. A new piece of information emerges: the 20 pupils in the class represent a whole, while the 13 pupils who eat at the canteen and the 7 pupils who eat at home are parts. We notice that table cells are not just numbers but numbers related to a situation.

Table 1*The situation box*

Pupils	20
	Whole

1The 13 cubes represent the 13 pupils who eat at the canteen, the 7 cubes represent the 7 pupils who eat at home. The cubes are placed under the number line with the numbers 0, 13 and 20.

Pupils-canteen	Pupils-home
13	7
Part 1	**Part 2**

As we can see, these representations are linked to the others. They provide different elements. For example, the number line illustrates the size of numbers (the 13-bridge is larger than the 7-bridge). The number box introduces generic words such as part-whole, which can then be used to talk about part-whole problems (combine problems, Riley et al., 1983). The teacher then shows how these representations can be related to each other and to the text of the situation, in a process of translating representations. The numbers of pupils-home and canteen can then be designated by different representations. The teacher links the representation by cubes (a cube is a pupil, 13 cubes are the 13 pupils eating at the canteen) and the number line (the bridge of 13 is the 13 pupils eating at the canteen...). In the box, part 1 (bottom left) is the 13 pupils eating at the canteen. This translation process makes it possible to introduce the mathematical writings $20 = 13 + 7 \Leftrightarrow 20 = 7 + 13 \Leftrightarrow 20 - 13 = 7 \Leftrightarrow 7 = 20 - 13 \Leftrightarrow 20 - 7 = 13 \Leftrightarrow 13 = 20 - 7$, a new form of representation directly linked to the other representations (the 13 are the pupils in the class without the pupils who eat at home). These representations show different properties of numbers.

The students then try to imitate what has been done, more or less identically (each student repeats alone what has just been done), then by modifying one element (the same unproblematic situation the teacher has just done with different numbers or another situation with the same numbers) then by modifying the situation (while keeping the same structure). The "worked example" gives a crucial place to the notion of analogy (Hofstadter & Sander, 2013). Each of the students' suggestions is then shown to others. Imitation, based on these representations, is organized along a broad gradient, from replicative imitation to creative imitation (Heyes, 2016, 2023).

c) The trio game

The question of problem posing is part of the more general question of problem solving. Problem posing is considered to promote problem solving (English, 1998; Silver, 1994; Yang & Cai, 2016). Cai and Hwang (2020) define problem posing as follows: students pose mathematical problems based on given problem situations, which may include mathematical expressions or diagrams, or students pose problems by modifying existing problems. The difficulties encountered in the classroom then relate, in particular, to the posing mathematical problems that are irrelevant from a mathematical point of view or from the point of view of the stakes of knowledge.

The first choice made within the research DEEC is the following. Based on the unproblematic situation, students in the classroom are asked to pose problems. The unproblematic situation will therefore be the basis for posing problems. Based on the unproblematic situation, it is possible to write different types of problems. A mathematical problem can be seen as "the search for the third number", which someone knows, the person who posed the problem. Let us look at the same example again. Using example 1 (there are 20 pupils in a class. 13 eat at the canteen. 7 eat at home), it is possible to pose three problems (finding the whole - finding the number of pupils in the class - or finding a part - finding the number of pupils who eat at the canteen, or finding the number of pupils who eat at home). Using example 2 (a pen weighs 8g. A notebook weighs 22g. The notebook weighs 14g more than the pen. The pen weighs 14g less than the notebook. The difference between the mass of the pen and the mass of the notebook is 14g), we can pose three problems (finding the large compared, finding the small compared and finding the difference). But the first implementation was difficult. Students spent a lot of time to write their three problems (if they succeeded) and they forgot the aim of the problems. A new game was introduced thanks to the engineering dialogue, the trio game. In this game, the unproblematic situation is represented by a "box of numbers". The box contains 3 numbers.

The aim is for the students to understand the relationships between units of account: seeing 20 pupils in the class as 13 pupils who eat at the canteen and 7 pupils who eat at home, or 13 as the pupils in the class without the pupils who eat at home. Using the box, see 20 as the sum of 13 and 7 or 13 = 20-7. The trio of numbers (20, 13, 7) in the situation box enables this work on the relationships between numbers. The trio of numbers changes status when one of the numbers is hidden, taking the form (20, x, 7), for example (see table 2).

Table 2

A new box

Pupils 20	
Whole	
Pupils-canteen	Pupils-home
x	7
Part 1	**Part 2**

This trio of numbers (20, x, 7) then becomes a model of the world that leads to calculation practices (x= 20-7, for example), either in the world of the students in the class, or in another world. The trio of numbers also becomes a model for anticipating actions. Under the "x", the student knows that there is a number, and that this number can be found in different ways. The "x" is seen as the unknown number, the solution to the problem situation. The use of the "x" by the students does not seem to be problematic, any more than the question mark or an blank space.

This trio then encourages the student to give an account of the unproblematic situation that has become a problem. For example, the teacher shows and presents the problem based on the representation: "There are 20 pupils in the class. There are 7 pupils who eat at home. How many pupils eat at the canteen?". The students can then imitate what the teacher has just done, identically (the student presents the problem alone), then by changing one element (the same situation with different numbers or another situation with the same numbers), then by changing the situation (while keeping the same structure). In this way, the trio game leads to a progressive understanding of mathematical problems, in a process of integration on the part of the students. This presupposes that the teacher organizes the system appropriately. The students can work out the relationship between the numbers in the box, the numbers designating elements of the problem situation.

d) The categorization

Each category of additive and multiplicative problems is worked on in the same way. An unproblematic situation is presented (e.g. the pupils in the class eating at the canteen or at home): this is an opportunity for the students to understand the words in the situation and to reformulate them in different ways (without mathematical problems). This unproblematic situation is presented in different ways (the box, the number line, mathematical writings). At the same time, the vocabulary specific to this type of problem is introduced (e.g., part 1, part 2, the whole). These representations are then used in different situations. Finally, the trio game is used to introduce the problem category (in this case, combine problems) by introducing the unknown number x into the box. The vocabulary already encountered (part or whole) is reused in the problems (e.g., we are looking for a part, we are looking for a whole). The mathematical writings are also repeated with the unknown number x. As before, the unproblematic situation is used as a basis for three problems. Then the other unproblematic situations (close to the initial situation) become the basis for posing new problems. All these situations are recognized as combine problems because they look like the problem of the students in the class who eat at the canteen or at home. Therefore, these problems can be solved.

Finally, when a new situation arises in the posing/solving problem, the student is expected to see this new situation as a combine problem or as the problem of the students in the class who eat at the canteen or at home. We could summarize this organization of the posing and solving problem as follows: at each stage there is a new problem. At the same time, the new problem (the unknown) can be adequately understood thanks to what has been done before (the known). This structure is repeated for each category of additive or multiplicative problems.

To illustrate this structure and the teacher's role in understanding it in class, we will now present what it happened in a class.

e) Description and analyze of a moment in a classroom

Here we show how a teacher and students work together to pose new problems. In a second-grade class (7-years-old), the students and the teacher worked on a new unproblematic situation: "In the class there 20 pupils. There are 12 pupils-girls and 8 pupils-boys" (see fig. 5 & 6).

Figure 5

On the white board

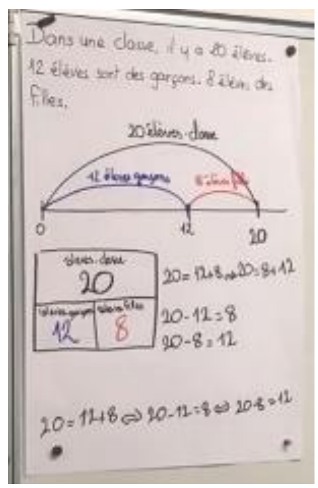
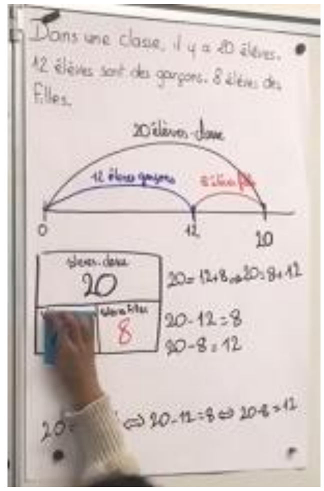


Figure 6

Hide a case



We can see that this new unproblematic situation is close to the initial situation we presented earlier. The teacher chooses to keep the units, i.e. pupils, pupils-girls and pupils-boys, in the same way as in the initial situation, class pupils, pupils-canteen, pupils-home. We can say that she uses the same category jargon. The teacher changes the situation. A student comes to place the mask, the x. He chooses to hide the number of boy pupils by placing a blue cache. The teacher then asks the question: “what do I want to find?” As we have said, posing a problem means asking a question, which the poser of the problem knows how to solve. In the classroom, the students seem to be hesitating. They propose different answers: the pupils-boys, all the pupils. During this hesitation, the teacher refers to the box and the number line, which she redraws (see fig. 7). She concludes: “If the person to whom you give the problem finds 12, he or she will have succeeded” and shows it.

Figure 7

The hands to show

We can say that problem posing is based on imitation at different levels: the unproblematic situation, the use of the cache, the question whose answer is known by the problem poser. Based on these elements, students can say the three expected problems (search for the whole or search for the part 1 or search for the part 2). They can also write them down on their own slate.

As we have presented, each category of additive and multiplicative problems is worked on in the same way. And at the same time, it is necessary to work on these categories in relation to each other. To enable students and teachers to build on the work done in each category, the collective proposed working on a tool, the repertoire instrument. It is *rendering present again* all what is done in the classroom. At first, we present this tool. After, we show its development. Then we explain the role of the collective in its development. Finally, we show two examples of its use by students.

3.2. A tool: the repertoire instrument

3.2.1. Presentation

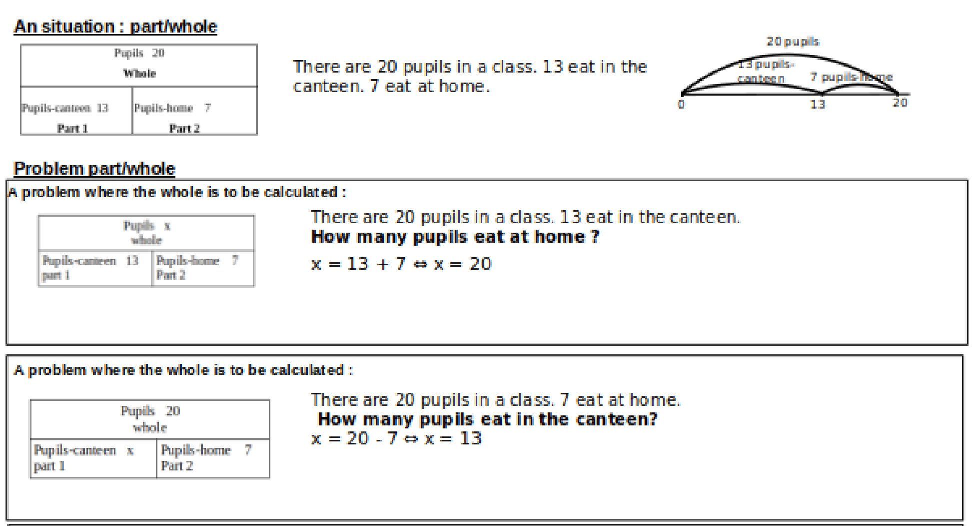
The repertoire instrument takes the form of a document containing unproblematic situations and a number of problematic situations accompanied by solutions to the problems posed. Its organization is complex, and closely linked to its function. Research has shown the role of *institutionalization* (Brousseau, 1997). It makes it possible to clarify the knowledge that can legitimately be referred to, and to recognize the usefulness of certain knowledge. This final recognition of the progress made by the pupils involves an initial recognition, in the day-to-day activity, of the knowledge that emerges. This is the process of institutionalization (*ibidem*), whereby the teacher assures the pupils that their activity has enabled them to rediscover legitimate knowledge outside the institution of the classroom, and makes them accountable for this knowledge from now on (Sensevy, 2014). The idea of institutionalization is thus developed: it produces a *seeing-as* (Wittgenstein, 1997), and is characterized by a style of thinking (Fleck, 2008). The repertoire instrument makes it possible to develop *a seeing as* through the analogy that students can draw between an old situation (known) and a new situation (unkown) that they have to work on. The analogy thus takes on an important and explicit role. For example, the unproblematic situation of pupils-canteen and pupils-home becomes the situation of part-whole problems (three kinds of problems, searching for the whole, searching for part 1 and searching for part 2). They are then designated as emblematic examples, which are grouped together in this document, the repertoire instrument. The organization of this repertoire instrument is complex. On the one hand, it gives an account of a category of problems based on emblematic examples (the category of combine problems, the category of comparison problems, the category of transformation problems, for additive problems). Secondly, it highlights the sub-categories identified in the emblematic examples (in the category of part-whole problems, the unknown is a part or the unknown is the whole; in the category of comparison problems, the unknown is the large compared, the small compared or the difference; in the category of transformation problems, it is necessary to find out whether it is a positive or negative transformation, and whether the unknown is the initial state, the final state or the transformation). As well as allowing these categorizations, the repertoire instrument enables students and teachers to identify the nature of the mathematical problems. The repertoire instrument therefore acts as a memory of what has been done in class (repertoire function). It also acts as a tool for creating and solving problems (instrument function). The repertoire instrument seems to be an essential support in posing/solving problems. As long as the teacher provides adequate support and the jargon is used.

3.2.2. A development over time

When a situation is worked on, a new page of the repertoire instrument is created. We can imagine that a teacher can say "Today, we worked on this". For example, based on the unproblematic situation, two problems are worked (see fig. 8).

Figure 8

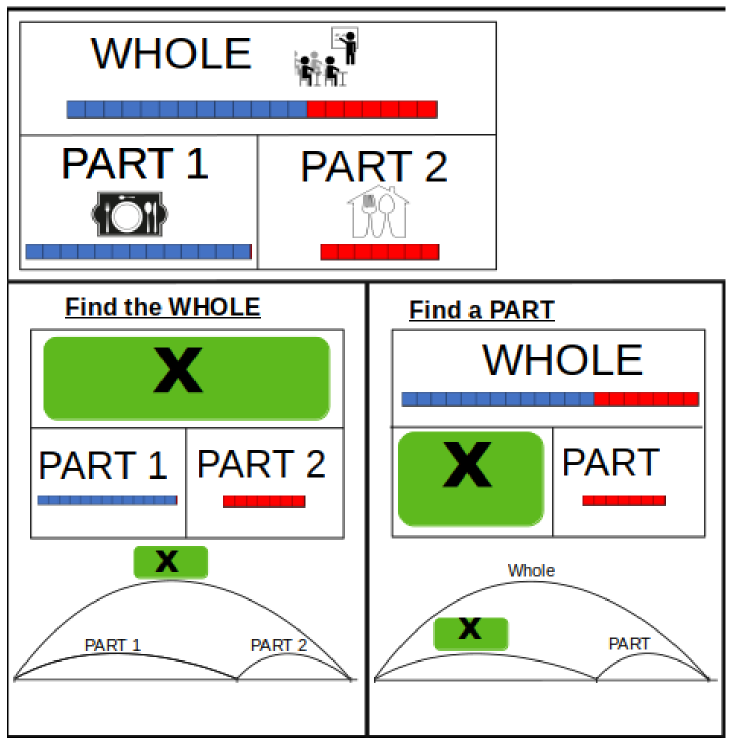
An unproblematic situation and two problems



It becomes possible and interesting to give up some details of the studied situations. This new repertoire instrument shows only the structure, and only a few words. The unproblematic situation based on the situation box with the cubes' representation (without number) and two problems based on this situation (without number), represented in the number box and the number line (see fig. 9). This new repertoire instrument shows a part-whole problem based on the situation of home- and canteen pupils. It could then be a tool for any work on creating part-whole problems involving quantities.

Figure 9

Part-whole problem in a glance

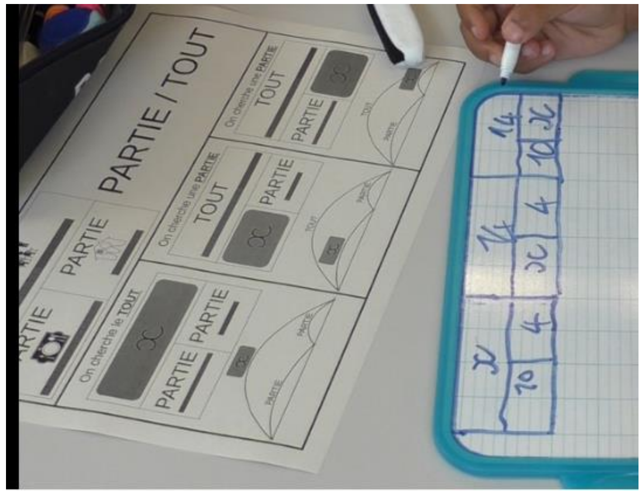


3.2.3. Examples in the classroom

In a first example, we want to show how a student is able to deal with the repertoire instrument. Thanks to this tool, he has to "pose" three new problems (see fig. 10). As we said before, he was asked to write three boxes, which are based on the same unproblematic situation he had chosen: the whole is 14, and the parts are 10 and 4. The trio game will help him to solve these problems by translating these boxes into texts such as “There are 14 pupils in the class. 4 pupils eat at home. How many pupils eat at the canteen?” in an imitative way.

Figure 10

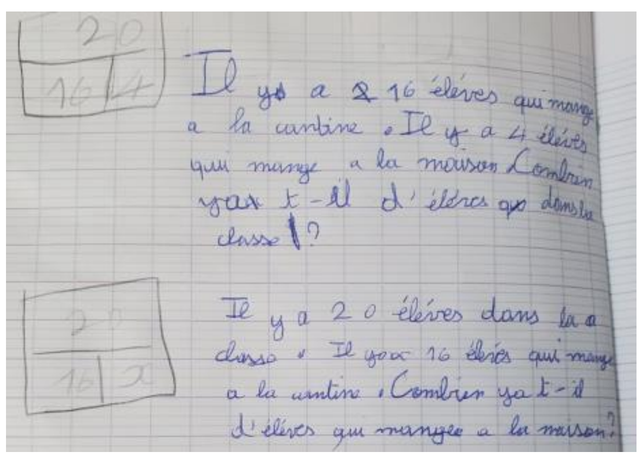
The repertoire instrument next to the slate



In the classroom, after this first imitation, the students are asked to write three problems based on the same situation or not. They are not asked to solve them. We can see from this work that pupils are able to deal with the three numbers even though they are not written. Students chose their own situations (see fig. 11 & 12).

Figure 11

Some problems written by pupils



There are 16 pupils who eat at the canteen. There are 4 pupils who eat at home.

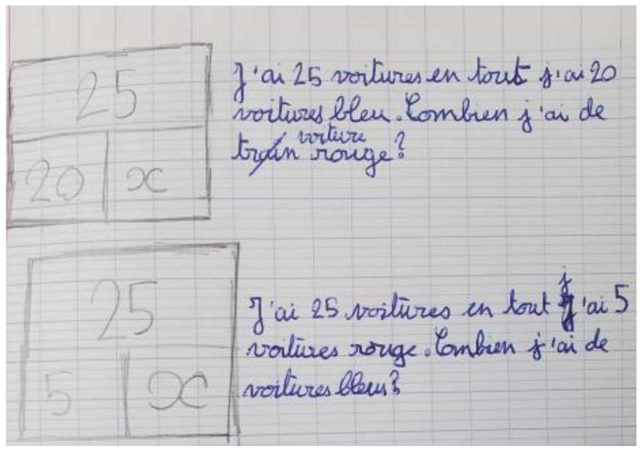
How many pupils are in the class?

There are 20 pupils in the class. There are 16 pupils who eat at the canteen.

How many pupils eat at home?

Figure 12

Some other problems



I have got 25 cars. I have got 20 blue cars.

How many red cars do I have?

I have got 25 cars. I have got 5 red cars.

How many blue cars do I have?

Depending on their understanding of the situation, each student will be able to pose problems, based on their habits of the situation box.

Our second example takes place in another classroom. A student has just posed and solved a problem. The text written by the pupil is as follows: "In a class, 14 pupils. There are 10 pupils in CE1. How many pupils are there in CE2?". Finally, he proposes the following answer: "There are 4 pupils in CE2". We can say that the number box enables problems to be posed and solved (see table 3). In the example shown, we see the power of the box (the student poses problems and solves them), but at the same time, the expected calculation is not at all present the expressions proposed at the beginning (20 = 13 + 7 ⇔ 20 = 7 + 13 ⇔ 20 - 13 = 7 ⇔ 7 = 20 - 13 ⇔ 20 - 7 = 13 ⇔ 13 = 20 - 7), are not taken up.

Table 3

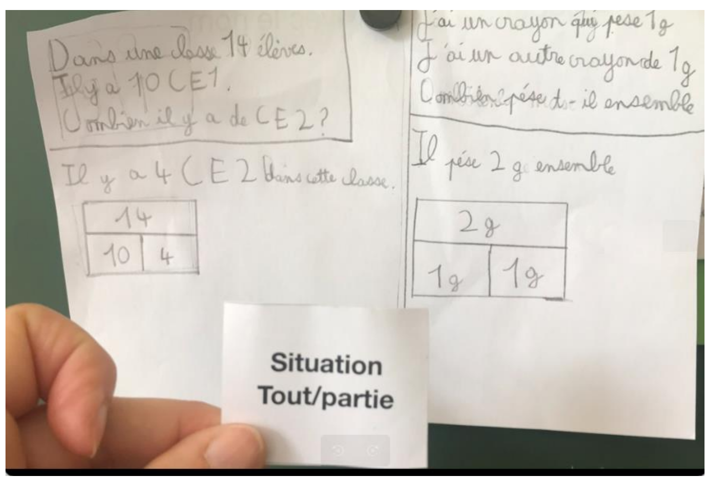
*Another example*3

Pupils	14
Whole	
Pupils-CE1	Pupils-CE2
10	4
Part 1	**Part 2**

A few moment later, the same student is asked to produce a new problem in the same category (part-whole), with an other unit, the mass. At first, he explains he is not able to deal with that question. The teacher helps him to remember a previous unproblematic situation, which is in the repertoire instrument ("A notebook weights 12g. A pen weights 6g. A notebook and a pen weight 18g together"). As soon as he reads the problem about mass, he becomes able to pose a new problem, without further help (see fig. 13).

Figure 13

Two problems



First problem (on the left)

In a class, 14 pupils. There are 10 pupils in CE1. How many pupils are there in CE2?

There are 4 pupils in CE2.

Second problem on the right)

I have got a pen which weight 1g. I have got an other pen whose weight is 1g. How do they weight together?

They weight 2g together.

In these both examples described briefly, we can see the instrument function of the repertoire instrument: the student uses the part-whole category to write new number boxes and pose new problems, more or less similar to the initial situation. We can notice in the second example that the teacher's role is to focus the student's attention on the object being worked on. In both examples students have habits to pose (and solve) problems on their own.

3.2.4. The repertoire instrument: design of a presentation's document

After presenting this tool, we want to present some features of this tool. To do so, we will explain the next step of this project. We want to share this tool with teachers who don't belong to the cooperative engineering collective. To do so, the research team drew up a document to explain the repertoire instrument and its use in the various units. We cannot describe and analyze all the discussions about this tool. In this paper, we simply look back at some of the engineering dialogue that took place during a meeting of the group. This is followed by a discussion *via* a discussion list. The teachers in the team present the different repertoire instruments designed in their classes (see fig. 8).

Firstly, a discussion on the function of the repertoire instrument was initiated at the collective meeting:

- It helps students to understand situations;
- It poses problems;
- It solves them;
- It is a way of exchanging ideas with students.

The need to offer students two levels of repertoire instrument was then highlighted. The first level refers to a situation, and is emblematic of a situation worked on by the pupils. The second level, which is more complex and should not be introduced too early, is categorical.

After this meeting, an exchange began on the mailing list about the presence of different representations (e.g. of mathematical writing), and the place of emblematic and categorical repertoire instrument. This dialogue was made possible by a shared background among engineering members. Indeed, the design of a document explaining this device to experimental classes requires in-depth knowledge of the organization of the sequence (trio game...), of the problem categories and of the *in situ* use with pupils of this device, as instrument and repertoire.

We want to show that design throughout the same category.

For example for part-whole problems, the repertoire instrument consists first of all of the emblem representation of this category. The number box is completed in reference to the situation that the students worked in class (home/canteen situation, see table 4).

Table 4

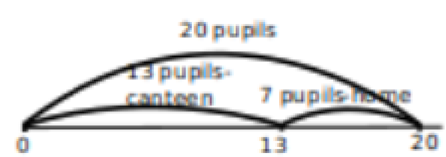
Representations[2]

Pupils 20	
Whole	
Pupils-canteen	Pupils-home
13	7
Part 1	**Part 2**

Then other representations come to translate this situation including mathematical writings (see fig. 14 and fig.15).

Figure 14

The number line



2 Specific representation of thepart- whole category applied to the emblematic situation

Figure 15

The mathematical writings

$$20 = 13 + 7 \leftrightarrow 20 = 7 + 13 \leftrightarrow 20 - 13 = 7$$
$$\leftrightarrow 7 = 20 - 13 \leftrightarrow 20 - 7 = 13 \leftrightarrow 13 = 20 - 7$$

Finally, the generic representation of this problem category is proposed (see table 5).

Table 5

The specific generic representation of the part-whole category[3]

Whole	
...	
Part 1	Part 2
...	...

The collective has designed a first situation, the pupils eating at home and the pupils eating at the canteen. This situation began with the unproblematic situation and the problems. These problems belong to the combine problems category. At the same time they represent this category: another combine problem is *the same as* the problem with pupils-home and pupils-canteen. This system needs to be put to practical use. For example, a new repertoire instrument was written in a class without the situation, but with some clues (the x, but not the situation of students). The repertoire instrument therefore serves as a memory of what has been done in class (repertoire function). It also serves as a tool for posing and solving problems (instrument function, example 1 and 2, above).

4. Discussion and conclusion

We would like to come back to the two recurrences we presented: the recurring structure in the sequence and the recurring structure in the elaboration of the sequence. The DEEC sequence is characterized by its recurring structure from one session to the next (see 3.1.1). Each session is organized as follow (see 3.1.2): an unproblematic situation as a whole class (see

3 Applicable to all situations in the part-whole category

a), the worked example and imitation work (see b), the trio game and problem posing and solving (see c), the categorization (see d). During these sessions, the teacher and pupils use the repertoire instrument (see 3.2). For example, additive problems are introduced with combine problems. They are implemented again with comparison problems, positive transformation problems and, finally, negative transformation problems. We have described the system developed within cooperative engineering, the sequence (see 3.1) and some actions that take place during the sequence (see 3.2). These actions are the result of the work carried out within the cooperative engineering project and its various iterations. As the sequence itself evolves, so do the teachers' actions (see 3.2.3).

We now remind you the second recurrence. The sequence has been developed within cooperative engineering: it has been written, implemented, analyzed and modified by the members of cooperative engineering, in an engineering posture. This implies a deep understanding of each step. The teacher and students establish habits of problem posing/solving based on representations, such as the number box, or word representations such as this problem represents the category of problems. To develop this sequence, the engineering dialogue between teachers and researchers relies on different symbolic representations (e.g. the number line, the box, the mathematical writings), whether linguistic (the categorization of mathematical problems) or tool (the repertoire instrument).

This sequence is structured to foster a shared understanding of the mathematical problems within the class thanks to these both recurrences. At first the students are able to imitate the teachers. After a while, they become good problem posers and solvers. For example, these two problems (see fig. 12) are well posed and well solved (Problem 1: I have 25 cars. I have 20 blue cars. How many red cars do I have? Problem 2: I have 25 cars. I have 5 red cars. How many blue cars do I have?). The search for the whole or a part is solved by everyone.

At the same time, the result of this repeated implementation and analysis is the development of the repertoire instrument. The repertoire instrument is a tool that reports on a shared understanding in the collective of teachers and researchers. As we have shown, the sequence is structured to foster a shared understanding of the problems within the class. This could be seen as a virtuous circle between engineering work and work in the classroom, a circle that contributes to build cultural evidence in education (Sensevy, 2021). The more we understand, the more the practice is effective. The more the practice is explicit, the more we can understand. The repertoire instrument could be seen as a clue to express this share understanding et the work done in the class.

This repertoire instrument which makes this shared understanding possible and visible is one of the consequence of the both recurrences too.

The DEEC research is a project over three years. The next one is to broadcast the sequence. Not only do teachers and researchers have to develop an shared understanding of the sequence, but they must now make their understanding explicit to other teachers. It is not simply a question of sharing the sequence, as a treatment that would produce results for students. It is about profoundly modifying teachers' understanding of the knowledge at stake in the problem posing and solving sequence, so that they would implement it with the same density of knowledge as the members of cooperative engineering. To get closer to this, the research is about a new step. The collective of teachers and researchers have written a text (as usual), a face-to-face support in meetings (less usual). Teachers and researchers point out some features of the practice. In the beginning, the student's work is based on imitation (see fig. 11). And it becomes more and more distant (see fig. 12). At the same time, they become aware of the categories that lead them to use a common mathematical jargon in the classroom. At each step in the sequence, the teacher and students can be supported by the previous step. For example, if a student is not able to pose a new problem, he can use the repertoire instrument, or he can draw a new box with three numbers... In this way, the students become better at posing and solving problems.

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